Effects of Radiation Forces on the Attitude of an Artificial Earth Satellite

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The motion of an artificial earth satellite about its center of mass as a result of radiation forces is investigated The satellite is assumed to be symmetrical, both geometrically and dynamically The sources of radiation considered are direct solar radiation, solar ıadiation reflected by the earth and its atmosphere, and direct radiation from the earth For these three cases, the forces and torques acting on an arbitrarily shaped satellite are derived The ıesults are in integral form and are dependent upon the satellite's surface geometry, its orientation relative to the earth and sun, and its mass distribution The effects of the radiation torques upon the otherwise unperturbed motion of the satellite about its center of mass can be divided into two motions: motion of the axis of symmetry about the satellite's angular momentum vector (relative to its center of mass), and motion of the angular momentum vector For the sources considered, the motion about the angular momentum vector is an unperturbed Euler-Poinsot motion, and the angular momentum vector itself, while remaining practically constant in magnitude, precesses and nutates relative to the earth-sun line for direct solar radiation and relative to a perigee coordinate system for reflected solar and diıect earth radiation

Nomenclature

A,B,C = principal, central moments of inertia of the satellite
c = speed of electromagnetic radiation
dA = element of surface area
$\mathbf{e}$ = unit vector parallel to $\mathbf{r}$
$\mathbf{e}_s$ = unit vector directed from center of earth toward sun
$\mathbf{e}'$ = unit vector directed from center of mass of satellite toward sun
f = true anomaly of the satellite in its orbit
$\mathbf{F}_1,\mathbf{F}_2,\mathbf{F}_3$ = force due to direct solar, reflected solar, and direct earth radiation
$\mathbf{H}$ = angular momentum of satellite about its center of mass
I = radiation intensity
J = radiation flux
$\mathbf{M}_1,\mathbf{M}_2,\mathbf{M}_3$ = moment about satellite s center of mass due to direct solar, reflected solar, and direct earth radiation
$\mathbf{n}$ = unit normal to element of surface area
$\mathbf{p}$ = position of center of pressure relative to center of mass
$\mathbf{r}$ = position of satellite relative to center of earth
$\mathbf{r}'$ = position of element of satellite's surface area relative to its center of mass
$\bar{R}$ = radius of the sphere taken as model for the earth and its atmosphere
R_e = average radius of earth
R_{e-S} = mean earth-sun distance
R_{s-S} = satellite-sun distance
S = solar constant at mean earth sun distance
s = satellite s spin
T = absolute temperature
V = potential of the radiation moments
X,Y,Z = perigee coordinate system
x,y,z = satellite's principal axes; z is axis of symmetry
δ = shadow factor; unity when satellite is in sunlight, zero when it is in the earth's shadow
ϵ = emissivity
θ = angle between z axis and $\mathbf{e}_s$
θ' = angle between z axis and $\mathbf{e}$
ρ,α,τ = reflectivity, absorptivity, and transmissibility
$\bar{\rho}$ = average reflectivity of the earth and its atmosphere
σ = Stefan-Boltzmann constant
ϕ = angle between $\mathbf{H}$ and z axis of satellite
$\boldsymbol{\omega}_e$ = angular velocity of $\mathbf{e}$ relative to inertial space
$\boldsymbol{\omega}_0$ = angular velocity of satellite's center of mass about the earth
$\boldsymbol{\omega}_p$ = angular velocity of (X,Y,Z) relative to inertial space
$\boldsymbol{\omega}$ = angular velocity of satellite about its center of mass

Subscripts

f = a derivative in inertial space
r = a derivative in the satellite body axes (x,y,z)

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Introduction

THE purpose of this paper is twofold First, the forces and torques acting on an arbitrarily shaped artificial earth satellite as a result of direct solar radiation, solar radiation reflected by the earth and its atmosphere, and direct earth radiation are determined Secondly, the effects of these radiation torques upon the motion of the satellite about its center of mass are analyzed, specific configurations being used as examples The radiation effect has been noted by Roberson,[1] who lists some order-of-magnitude estimates Holl,[2] considering direct solar radiation only, assumes that the resulting force acting on the satellite can be expressed in the form $F = p_0AC_f$ (A = projected area, p_0 = radiation pressure in the vicinity of the eaıth) and shows that the "radiation force coefficient" C_f is found to be within the limits $0 < C_f \leq 2$ for several convex shapes McElvain[3] derives analytical expressions for the force and torque acting on an

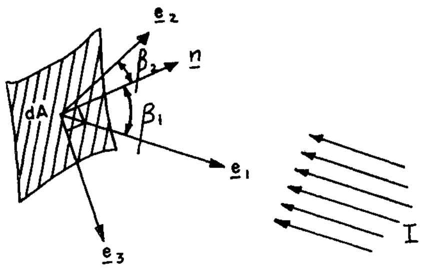

Fig 1 Surface geometry for incident and reflected radiation

arbitrarily shaped body due to direct solar radiation and determines, for two particular satellite shapes, the change in the satellite's angular momentum necessary to maintain a specified vehicle orientation There has also been some analysis of the use of radiation forces for satellite stabilization, such as that given by Sohn[4] and Newton[5] A related problem of thermal radiation incident upon an earth satellite has received considerable attention A few of those who have studied the problem are Cunningham,[6, 7] Katz,[8] Mark and Ostrach,[9] and Wood and Carter[10] The chief sources of radiation which cause forces and possibly torques on an artificial earth satellite are direct solar radiation, solar radiation reflected by the earth and its atmosphere, direct radiation from the earth and its atmosphere, and radiation emitted by the satellite The analysis of the last case requires a knowledge of the temperature in the satellite's shell, and, in general, it is a much smaller effect that the first three cases There are special situations, however, when this effect may be comparable to the other three; for example, the pressure experienced by emission from a "black" flat plate receiving solar radiation, and thus emitting an equal amount of radiation, is equal to two-thirds of the pressure due to direct solar radiation In general, the satellite does not act as a "blackbody" (it will reflect as well as conduct portions of the incident energy), and, furthermore, it emits radiation in all directions, thus tending to produce no resultant force For these reasons, this effect will be neglected in this paper

The first three sources of radiation are all exterior to the satellite and are covered by the following general example From Fig 1, the incoming radiation of intensity I (parallel to the unit vector $\mathbf{e}_1$) makes an angle β_1 with the normal to the surface element dA which has radiation properties ρ, α, and τ (the reflectivity, absorptivity, and transmissibility, respectively) The radiation, after striking the surface, is partly transmitted through the material, partly absorbed by the material, and partly reflected by the surface The reflected radiation (parallel to the unit vector $\mathbf{e}_2$) makes an angle β_2 with the surface normal The unit vector $\mathbf{e}_3$ perpendicular to $\mathbf{n}$ and coplanar with $\mathbf{n}$, $\mathbf{e}_1$, and $\mathbf{e}_2$ can be defined by the equation

$$\mathbf{e}_3 = \mathbf{n} \times (\mathbf{e}_1 \times \mathbf{n}) / \sin\beta_1$$

If the surface is assumed to be a specular reflector, geometric optics yields $\beta_1 = \beta_2 = \beta$ The incident radiation produces a force, J being the radiation flux and c the velocity of light,

$$d\mathbf{F}_i = -(\alpha + \rho)(J/c)dA\ \mathbf{e}_1$$

whereas the reflected radiation produces a force

$$d\mathbf{F} = -\rho(J/c)dA\ \mathbf{e}_2$$

Thus, the total force on the element of area dA is

$$d\mathbf{F} = -(J/c)dA[(\alpha + \rho)\mathbf{e}_1 + \rho\ \mathbf{e}_2]$$

or, in terms of the unit vectors $\mathbf{n}$ and $\mathbf{e}_3$,

$$d\mathbf{F} = -(J/c)dA[(\alpha + 2\rho)\cos\beta\mathbf{n} + \alpha\mathbf{n} \times (\mathbf{e}_1 \times \mathbf{n})] \quad (1)$$

In general, the radiation flux J depends on the shape, temperature, and emissivity of the radiation source as well as the position of dA relative to it Similarly, ρ, α, and τ for the receiving surface depend on the temperature of dA, the wavelength of the incident radiation, and the angle of incidence It is also possible that the surface properties (including ρ, α, and τ) of a particular satellite may change with time as a result of external factors Such a change in surface composition has been noted in the first Soviet satellite by Yatsunskii and Gurko[11]

In this analysis, ρ, α, and τ will be assumed constant for solar radiation, both direct and earth-reflected, whereas for the earth's radiation a different set of values will be taken (ρ', α, τ'), but they also will be assumed constant The other assumptions made in this analysis are as follows: 1) the tranmissibility of the satellite's surface, τ, is zero; 2) the satellite has both dynamic and geometric symmetry (z axis is the axis for both), and its surface, a specular reflector, is everywhere convex; 3) the solar radiation reflected by the earth and its atmosphere can be represented by reflection from a sphere of radius $\bar{R}$ with reflectivity $\bar{\rho}$; and 4) direct radiation from the earth is not affected by the earth's atmosphere The last two assumptions are made to circumvent the problem of scattering and absorption of radiation by the earth's atmosphere This is a separate problem in itself, and there is no intention of considering it in this analysis

From Fig 2, the radiation flux at dA_2 due to unifoım, diffuse emission from the surface element dA_1 is

$$dJ_{2\,1} = (\epsilon_1 \sigma T_1^4/\pi L^2)\cos\beta_1 \cos\beta_2 dA_1 \quad (2)$$

in which T_1 is the temperature and ϵ_1 the emissivity of dA_1 The Stefan-Boltzmann constant is represented by σ The total radiation flux at dA_2 due to body 1 is

$$J_{2\,1} = \int_{A_1*} \left(\frac{\epsilon_1 \sigma T_1^4}{\pi L^2}\right) \cos\beta_1 \cos\beta_2 dA_1$$

where the integration is carried over the surface area of body 1 which is "seen" by the element of area dA_2, that is, over all elements of area of body 1 such that $0 \leq \beta_1 \leq \pi/2$ and $0 \leq \beta_2 \leq \pi/2$ In these formulas, L is the straight line joining the area elements dA_1 and dA_2, the normals to which are inclined at angles β_1 and β_2, respectively, to it

On the basis of Eqs (1) and (2) and the stated assumptions, it is now possible to write the total force acting on an element of surface area of an earth satellite as a result of the three sources of radiation From Fig 3, the spherical model of the earth and its atmosphere being shown as the earth itself,

$$d\mathbf{F} = d\mathbf{F}_1 + d\mathbf{F}_2 + d\mathbf{F}_3$$

where

$$d\mathbf{F}_1 = -\delta(S'/c)[\cos^2\eta_2(1 + \rho)\mathbf{n} + \cos\eta_2(1 - \rho)\mathbf{n} \times (\mathbf{e}'' \times \mathbf{n})]dA \quad (3)$$

is the force due to direct solar radiation The shadow factor δ is zero when the satellite is eclipsed by the earth and unity otherwise; then

$$d\mathbf{F}_2 = -\delta \int_{\bar{A}e} \left(\frac{\bar{\rho}S}{c\pi L'^2}\right) \cos\eta_1 \cos\xi_1'[\cos^2\xi_2'(1 + \rho)\mathbf{n} + \cos\xi_2'(1 - \rho)\mathbf{n} \times (\mathbf{1}' \times \mathbf{n})]d\bar{A}\ dA \quad (4)$$

is the force due to solar radiation reflected by the earth and its atmosphere (reflected from a sphere of radius $\bar{R}$), and

$$d\mathbf{F}_3 = -\int_{Ae} \left(\frac{\epsilon_e \sigma T_e^4}{c\pi L'^2}\right) \cos\xi_1'[\cos^2\xi_2'(1 + \rho')\mathbf{n} + \cos\xi_2'(1 - \rho')\mathbf{n} \times (\mathbf{1}' \times \mathbf{n})]dA\ dA \quad (5)$$

is the force due to direct radiation from the earth The integration in Eq (4) is over the surface area of the earth-atmosphere model sphere seen by dA and not in the earth's shadow In Eq (5), the integration is over the surface area of the earth seen by dA The symbol S represents the solar constant at the earth, S' that at the satellite In general, the extent of these surface integrations will depend not only on the relative positions of the earth, sun, and satellite but also on the particular element of area dA

The unit vector from an element of the satellite's surface area to the sun, $\mathbf{e}''$, in Eq (3) can be replaced by the unit vector from the satellite's center of mass to the sun, $\mathbf{e}'$, since the dimensions of the satellite are negligible compared to r and R_{-s} For the same reason, and to simplify the integrations, each element of surface area dA can be moved parallel to itself to the center of mass, thus replacing L', $1'$, ξ_1', and ξ_2' with L, 1, ξ_1, ξ_2 in Eqs (4) and (5) (see Fig 3) Also note that S' can be related to S by

$$S' = S[1 + 2(r/R_{\ s})\mathbf{e}\ \mathbf{e} + 0(r^2/R_{\ s}^2)] \tag{6}$$

where $\mathbf{e}_r$ is the unit vector from the earth directed toward the satellite Because of this small difference between S' and S, the latter value will be used for the remainder of this analysis

Incorporating the preceding simplifications into Eqs (3–5), one obtains

$$d\mathbf{F}_1 = -\delta(S/c)[\cos^2\eta_2(1+\rho)\mathbf{n} + \cos\eta_2(1-\rho)\mathbf{n}\times(\mathbf{e}'\times\mathbf{n})]dA \tag{7}$$

$$d\mathbf{F}_2 = -\delta\int_{\bar{A}e}\left(\frac{S\bar{\rho}}{c\pi L^2}\right)\cos\eta_1\cos\xi_1[\cos^2\xi_2(1+\rho)\mathbf{n} + \cos\xi_2(1-\rho)\mathbf{n}\times(\mathbf{1}\times\mathbf{n})]d\bar{A}\,dA \tag{8}$$

$$d\mathbf{F}_3 = -\int_{Ae}\left(\frac{\epsilon_e\sigma T_e^4}{c\pi L^2}\right)\cos\xi_1[\cos^2\xi_2(1+\rho')\mathbf{n} + \cos\xi_2(1-\rho')\mathbf{n}\times(\mathbf{1}\times\mathbf{n})]dA\,dA \tag{9}$$

The total force acting on the satellite due to the three sources of radiation is

$$\mathbf{F} = \int_{A_1*} d\mathbf{F}_1 + \int_{A_2*} d\mathbf{F}_2 + \int_{A_3*} d\mathbf{F}_3 \tag{10}$$

where $A_1{}^*$ is the total surface area of the satellite seen by the sun, $A_2{}^*$ is the total surface area of the satellite seen by that part of the earth not in the earth's shadow, and $A_3{}^*$ is the total surface area of the satellite seen by the earth The total external moment about the satellite's center of mass is

$$\mathbf{M} = \int_{A_1*}\mathbf{r}'\times d\mathbf{F}_1 + \int_{A_2*}\mathbf{r}'\times d\mathbf{F}_2 + \int_{A_3*}\mathbf{r}'\times d\mathbf{F}_3 \tag{11}$$

where $\mathbf{r}'$ is the position vector of an element of area of the surface relative to the center of mass If the satellite's surface can be divided into a number of subsections, or if the satellite's reflectivity varies from section to section, Eq (11) may be replaced by suitable summation formulas

Equations (10) and (11) with Eqs (7–9) formally give the forces and torques acting on an earth satellite, under the stated assumptions, as a result of direct solar radiation, solar radiation reflected by the earth and its atmosphere, and direct radiation from the earth The surface integrations that are involved for an arbitrarily shaped satellite are very complicated (especially for the determination of $\mathbf{F}_2$ and $\mathbf{F}_3$), and, except for a few special cases, analytical evaluations of the integrals in Eqs (10) and (11), appear to be unobtainable The two papers by Cunningham,[6,7] on a related problem, indicate the nature of this difficulty

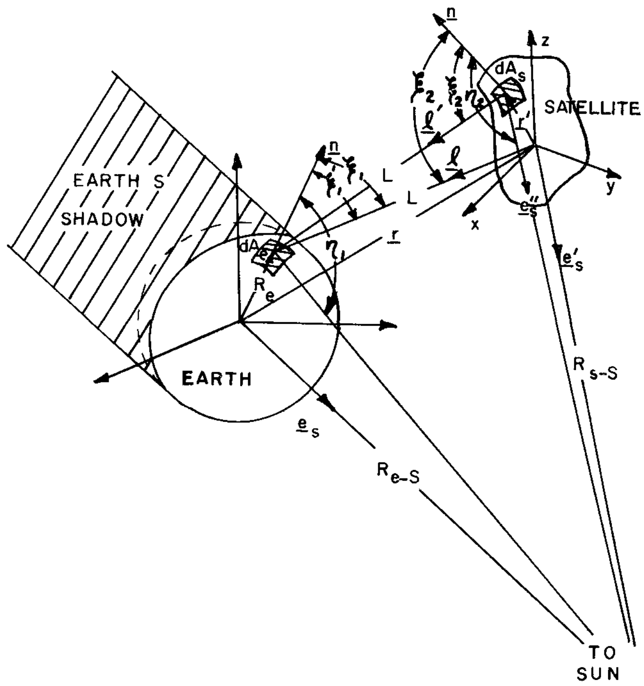

Fig 3 Geometry of receiving element dA_s of satellite relative to the emitting element dA_e of earth and relative to the sun

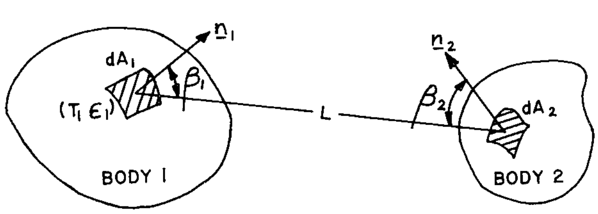

Fig 2 Radiation at element dA_2 due to emission from element dA_1

Effects of Torques upon Satellite Attitude

In general, there are many different perturbing forces and torques acting on an earth satellite, and the resulting motion will depend on all of these forces and torques These perturbations are usually small, relative to the unperturbed motion, and generally they are investigated separately, with the total motion being determined by adding the individual results In this analysis, the effects of radiation forces on the motion about its center of mass of an otherwise unperturbed satellite‡ will be investigated, and, assuming no coupling with the other perturbations, the results may be added to the already existing equations for the effects of the earth's gravity, atmospheric drag, etc , which have been analyzed by Beletskii,[12] Colombo,[13] and others Direct solar radiation contributes the major component of force, and it will be considered first

Direct Solar Radiation

From Eqs (10) and (7),

$$\mathbf{F}_1 = -\delta\left(\frac{S}{c}\right)\int_{A_1*}[\cos^2\eta_2(1+\rho)\eta + \cos\eta_2(1-\rho)\mathbf{n}\times(\mathbf{e}'\times\mathbf{n})]dA \tag{12}$$

where $A_1{}^*$ is the surface area of the satellite "seen" by the sun From Eq (11), the moment about the satellite's center of mass due to $\mathbf{F}_1$ is

$$\mathbf{M}_1 = -\delta\left(\frac{S}{c}\right)\int_{A_1*}\mathbf{r}'\times[\cos^2\eta_2(1+\rho)\mathbf{n} + \cos\eta_2(1-\rho)\mathbf{n}\times(\mathbf{e}'\times\mathbf{n})]dA_s \tag{13}$$

‡ The satellite s unperturbed motion, $\mathbf{M} = 0$, is characterized by $\mathbf{H}$ being constant in inertial space and by a regular precession of the axis of symmetry about $\mathbf{H}$

The unit vector $\mathbf{e}\,'$ appearing in Eqs (12) and (13) can be related to more useful vectors by referring to Fig 3 One finds

$$\mathbf{e}_s' = [1 + (r/R_{-s})\mathbf{e}\ \ \mathbf{e}_s + O(r^2/R_{e\text{-}s}^2)]\mathbf{e} - [(r/R_{e\text{-}s}) + O(r^2/R_{e\text{-}s}^2)]\mathbf{e}_r$$

For $r = 25{,}000$ miles, $r/R_{e\text{-}s} = 0\ 00027$ Thus to a very good approximation $\mathbf{e}_s'$ will be taken equal to $\mathbf{e}$

Before proceeding to particular satellite shapes, consider the following general analysis Since the only configurations under consideration are those with both geometric and dynamic symmetry, it would be plausible to assume $\mathbf{F}_1$ of the form

$$\mathbf{F}_1 = -\delta(F_1'\mathbf{e}\ + F_1''\mathbf{k}) \tag{14}$$

The first term, $-\delta F_1'\mathbf{e}_s$, would follow directly from Eq (12) if $\rho = 0$, whereas if $\rho \neq 0$ the combined result would apply, since $\mathbf{e}_s$ and $\mathbf{k}$ are the only preferential directions Also, because of the symmetry, the center of pressure will lie on the axis of symmetry, and the moment about the center of mass due to $\mathbf{F}_1$ will be

$$\mathbf{M}_1(\theta) = -\delta F_1'(\theta)p(\theta)\mathbf{k} \times \mathbf{e}_s \tag{15}$$

where $p\mathbf{k} = \mathbf{p}$, the position vector of the center of pressure relative to the center of mass, and F_1' are functions only of the angle θ between $\mathbf{e}_s$ and $\mathbf{k}$

The equation of motion about the center of mass is

$$\left.\frac{d\mathbf{H}}{dt}\right)_f = \mathbf{M}_1 \text{ or } \left.\frac{d\mathbf{H}}{dt}\right) + \boldsymbol{\omega}\ \times \mathbf{H} = \mathbf{M}_1 \tag{16}$$

with $\mathbf{H} = \bar{I}\ \boldsymbol{\omega}$, $\bar{I}$ being the inertia tensor of the satellite about its center of mass, and $\boldsymbol{\omega}_s = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega\,\mathbf{k}$, where $\boldsymbol{\omega}$ is the angular velocity of the satellite's principal axes (x,y,x) relative to inertial space From Eq (15) $\mathbf{M}_1\ \mathbf{k} = 0$, and from the assumed dynamic symmetry $\mathbf{i}\ \bar{I}\ \mathbf{i} = \mathbf{j}\ \bar{I}\ \mathbf{j} = A = B$ Thus, there is one integral of Eq (16):

$$\mathbf{H}\ \mathbf{k} = h_1 \quad \text{or} \quad \omega_z = s \text{ (const)} \tag{17}$$

The moment $\mathbf{M}_1$ has been assumed to depend only on the angle θ; thus, if

$$M_1(\theta) = -\partial V/\partial\theta \tag{18}$$

then

$$\tfrac{1}{2}\mathbf{H}\ \boldsymbol{\omega}\ - \mathbf{H}\ \boldsymbol{\omega}_e + V(\theta) = h_2 \tag{19}$$

which expresses conservation of energy in the rotating coordinate system (x,y,z) and provides an additional integral of the motion Because of the dynamic symmetry and the fact that $\boldsymbol{\omega}$ can be assumed constant, Eq (19) may be rewritten in the scalar form

$$H^2 - 2A\omega\ \cos\nu H + [2AH_0\omega\ \cos\nu_0 - H_0^2 + 2A(V - V_0)] = 0 \tag{20}$$

where H_0, V_0, and ν_0 are the initial (or unperturbed) values of H, V, and of ν, the angle between $\mathbf{H}$ and $\boldsymbol{\omega}$ Equation (20) is a quadratic equation for H which yields

$$H = H_0\left\{\frac{A\omega_e\cos\nu}{H_0} + \left[1 + \left[\left(\frac{A\omega_e\cos\nu}{H_0}\right)^2 - 2\frac{A\omega_e\cos\nu_0}{H_0} - \frac{2A}{H_0^2}(V - V_0)\right]\right]^{1/2}\right\} \tag{21a}$$

or

$$H = H_0\left\{1 + \frac{A\omega_e}{H_0}(\cos\nu - \cos\nu_0) - \frac{A}{H_0^2}(V - V_0) + O\left[\frac{A^2\omega_e}{H_0^2}\right] + O\left[\frac{A^2}{H_0^4}(V - V_0)^2\right]\right\} \tag{21b}$$

The positive sign has been taken for the square root, so that $H = H_0$ when $\nu = \nu_0$ and $V = V_0$ From Eq (21b), it is noted that there are no secular changes in H to order H_0^{-1} (M_1 will be periodic in θ, but V will not necessarily be so) At this point, two different problems must be distinguished The case

$$(A\omega\ /H_0)(\cos\nu - \cos\nu_0) - (A/H_0^2)(V - V_0) \ll 1 \tag{22}$$

corresponds to a satellite whose kinetic energy of rotation about its center of mass is large with respect to the work done by the external force Note that V is proportional to S/c (approximately $9\ 4 \times 10^{-8}$ psf) and ω is about 1 deg/day For this case, H may be taken equal to its unperturbed value H_0

The case

$$(A\omega\ /H_0)(\ \cos\nu - \cos\nu_0) - (A/H_0^2)(V - V_0) = 0(1)$$

corresponds to a satellite whose kinetic energy of rotation about its center of mass is not large as compared to the work done by the external force, for example (V being a small quantity), a satellite placed in orbit with an angular velocity close to ω In such a problem, the orbital motion of the center of mass and the motion about the center of mass are coupled, and a libration-type analysis would be necessary An analysis of this type was carried out by Beletskii[14] for the case of perturbing torques caused by the earth's gravity

In the present analysis the conditions of the first case are assumed, and, for a particular problem with given values of H_0, etc , the extremum values of Eq (22) may be computed and compared to unity From Eq (17), $\mathbf{H}\ \mathbf{k} = Cs = H\cos\phi$, where ϕ is the angle between $\mathbf{H}$ and $\mathbf{k}$ Thus, for $H = H_0$, $\phi = \phi_0$, its unperturbed value The precession rate of the axis of symmetry about the angular momentum vector $\dot\psi$ is given by $\dot\psi = H/A$ when the angular velocity of the plane of $\mathbf{H}$ and $\mathbf{e}_s$ is neglected relative to $\dot\psi$ (see Fig 4) So if $H = H_0$, $\dot\psi = H_0/A$, its unperturbed value Thus, there are no secular effects in the motion of the axis of symmetry about $\mathbf{H}$ Any secular effects in the motion must then be confined to the orientation of $\mathbf{H}$ in inertial space The angular momentum vector may be expressed as $H_0\mathbf{h}$, where, from Fig 4,

$$\mathbf{h} = \cos\mu\mathbf{e}\ + \sin\mu(\sin\lambda\mathbf{e}_1 + \cos\lambda\mathbf{e}_2) \tag{23}$$

where $\boldsymbol{\omega}_e = \omega\,\mathbf{e}_1$, and $\mathbf{e}_2 = \mathbf{e}\ \times \mathbf{e}_1$ Then, by Eq (16), one obtains

$$(d\cos\mu/dt)_f = -\omega\,\mathbf{h}\ \mathbf{e}_2$$

which, upon substituting Eq (23), becomes

$$(d\mu/dt)_f = \omega\ \cos\lambda \tag{24}$$

Accordingly, except for the effect due to the rotation of $\mathbf{e}$, the angle between $\mathbf{H}$ and $\mathbf{e}$ remains constant

From Eqs (16, 15, and 23),

$$-\delta[F_1'(\theta)p(\theta)/H_0]\mathbf{k} \times \mathbf{e}\ = (d\mathbf{h}/dt)f$$

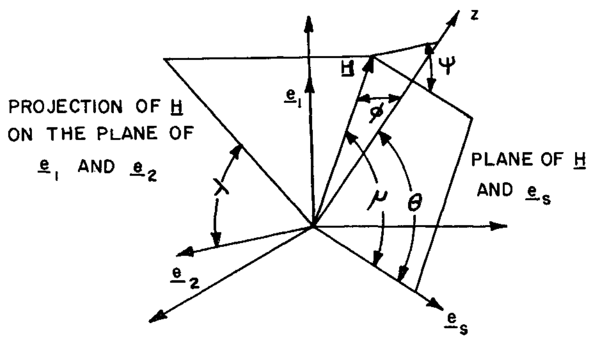

Fig 4 Orientation of angular momentum vector and axis of symmetry of satellite relative to e_s and inertial space

and

$$-\delta[F_1'(\theta)p(\theta)/H_0]\mathbf{k}\times\mathbf{e} = \{-\sin\mu\,\dot\mu\mathbf{e}_s + \cos\mu(d\mathbf{e}_s/dt)_f + \cos\mu\,\mu(\cos\lambda\mathbf{e}_s + \sin\lambda\mathbf{e}_1) + \sin\mu(-\sin\lambda\mathbf{e}_2 + \cos\lambda\mathbf{e}_2)\dot\lambda + \sin\mu[\cos\lambda(d\mathbf{e}_2/dt)_f + \sin\lambda(d\mathbf{e}_1/dt)_f]\} \quad (25)$$

Equation (25) reduces to three scalar equations:

$$-\sin\mu\,\dot\mu + \omega\,\sin\mu\,\cos\lambda = -\delta[F_1'(\theta)p(\theta)/H_0](\mathbf{k}\times\mathbf{e})\,\mathbf{e} \quad (26)$$

$$\sin\lambda\,\cos\mu\,\dot\mu + \sin\mu\,\cos\lambda\,\dot\lambda = -\delta[F_1'(\theta)p(\theta)/H_0](\mathbf{k}\times\mathbf{e})\,\mathbf{e}_1 \quad (27)$$

$$-\omega\,\cos\mu + \cos\mu\,\cos\lambda\,\dot\mu - \sin\mu\,\sin\lambda\,\dot\lambda = -\delta[F_1'(\theta)p(\theta)/H_0](\mathbf{k}\times\mathbf{e})\,\mathbf{e}_2 \quad (28)$$

Equation (26) is identical to the result already obtained, that is, Eq (24) Substituting Eq (24) into Eqs (27) and (28) leads to

$$\cos\lambda\,\dot\lambda = -\delta[F_1'(\theta)p(\theta)/H_0\sin\mu](\mathbf{k}\times\mathbf{e})\,\mathbf{e}_1 - \omega_e\cot\mu\,\cos\lambda\,\sin\lambda \quad (29)$$

$$\sin\lambda\,\dot\lambda = \delta[F_1'(\theta)p(\theta)/H_0\sin\mu](\mathbf{k}\times\mathbf{e})\,\mathbf{e}_2 - \omega\,\cot\mu\,\sin^2\lambda \quad (30)$$

These are two equations in the one unknown $\dot\lambda$ This redundancy arises because of the assumption that $\mathbf{H} = H_0\mathbf{h}$, when in fact the magnitude of $\mathbf{H}$ is not constant but is given by Eq (21) If $\mathbf{h}$ is to remain a unit vector, the velocity of $\mathbf{H}$ must be perpendicular to $\mathbf{H}$, and the axis of symmetry must therefore lie in the plane of $\mathbf{H}$ and $\mathbf{e}$, except for the special case $\mu = 0$ [This is easily seen from Eqs (29) and (30) by solving for $\dot\lambda$ and equating the two results] But it has already been shown that the axis of symmetry precesses about $\mathbf{H}$ with a rate $\dot\psi$, which is equal to H_0/A, whereas $\mathbf{H}$ precesses about $\mathbf{e}$ with a rate, which, from Eqs (29) and (30), is proportional to ω and $F_1'p/H_0$ The potential V is generally a very small quantity, and $\omega_e \ll H_0/A$ from Eq (22); thus $\dot\lambda \ll \dot\psi$, so that an average value for $\dot\lambda$ can be obtained from the preceding equations by averaging over a complete rotation of the z axis about the angular momentum vector Such an averaging process will eliminate the inconsistency in the equations for $\dot\lambda$ In similar analyses for determining the effects of other types of perturbations upon satellite attitude, this averaging is performed at the beginning of the formulation of the perturbing moments This, in effect, replaces the instantaneous moments by an average moment This method is employed by both Beletskii[12] and Colombo[13] One can easily show from Eqs (29) and (30) that the average precession rate is

$$\left\langle\frac{d\lambda}{dt}\right\rangle = \frac{\delta}{2\pi H_0\,\sin\mu}\int_0^{2\pi} F_1'(\theta)p(\theta)[(\sin\lambda\mathbf{e}_2 - \cos\lambda\mathbf{e}_1)(\mathbf{k}\times\mathbf{e})]d\psi - \omega\,\cot\mu\,\sin\lambda \quad (31)$$

where $F_1'(\theta) = F_1'[\theta(\psi)]$ and $p(\theta) = p[\theta(\psi)]$

Equation (31) may be rewritten as

$$\left\langle\frac{d\lambda}{dt}\right\rangle = -\frac{\delta}{2\pi H_0\,\sin\mu}\int_0^{2\pi} F_1'(\theta)p(\theta)[(\sin\lambda\mathbf{e}_1 + \cos\lambda\mathbf{e}_2)\,\mathbf{k}]d\psi - \omega\,\cot\mu\,\sin\lambda \quad (32)$$

and, from Fig 4,

$$\mathbf{k} = (\cos\phi\,\cos\mu + \sin\phi\,\sin\mu\,\cos\psi)\mathbf{e}_s + (\cos\phi\,\sin\mu\,\sin\lambda + \sin\phi\,\cos\lambda\,\sin\psi - \sin\phi\,\cos\mu\,\sin\lambda\,\cos\psi)\mathbf{e}_1 + (\cos\phi\,\sin\mu\,\cos\lambda - \sin\phi\,\sin\lambda\,\sin\psi - \sin\phi\,\cos\mu\,\cos\lambda\,\cos\psi)\mathbf{e}_2 \quad (33)$$

Since $\phi = \phi_0$, Eq (32) is

$$\left\langle\frac{d\lambda}{dt}\right\rangle = -\frac{\delta}{2\pi H_0}\int_0^{2\pi} F_1'(\theta)p(\theta)[\cos\phi_0 - \sin\phi_0\,\cot\mu\,\cos\psi]d\psi - \omega_e\,\cot\mu\,\sin\lambda \quad (34)$$

where μ is considered constant during the integration Thus the motion of the satellite under the influence of direct solar radiation can be described as an unperturbed motion (i e , no secular effects) of the axis of symmetry about the angular momentum vector, which itself remains practically constant in magnitude but precesses and nutates relative to $\mathbf{e}$ The average precession and nutation rates are given by Eqs (34) and (24), respectively If the rotation of $\mathbf{e}_s$ can be neglected, $\mathbf{H}$ performs a regular precession ($\mu = \mu_0$) about $\mathbf{e}$ with an average rate

$$\left\langle\frac{d\lambda}{dt}\right\rangle_0 = -\frac{\delta}{2\pi H_0}\int_0^{2\pi} F_1'(\theta)p(\theta)[\cos\phi_0 - \sin\phi_0\,\cot\mu\,\cos\psi]d\psi$$

If the satellite enters the earth's shadow ($\delta = 0$), the motion becomes the unperturbed motion ($\mathbf{M}_1 = 0$), and $\mathbf{H}$ is constant in inertial space at its value upon entering the shadow The general results that have been derived are now applied to spherical and cylindrical satellites §

The force due to direct solar radiation on a spherical satellite of radius R and with surface reflectivity ρ is

$$\mathbf{F}_1 = -\delta(S/c)A\,\mathbf{e}$$

where A_s is the projected area of the satellite as seen by the sun Thus, from Eq (14),

$$F_1'(\theta) = (S/c)A$$

If the separation distance p is a function of θ, $p(\theta)$, then

$$\mathbf{M}_1(\theta) = -\delta(S/c)A\,p(\theta)\mathbf{k}\times\mathbf{e}$$

$$V(\theta) = \delta(S/c)A\int p(\theta)\,\sin\theta\,d\theta$$

Therefore

$$d\mu/dt = \omega\,\cos\lambda$$

$$\left\langle\frac{d\lambda}{dt}\right\rangle = -\delta\left(\frac{S}{c}\right)\frac{A\,\cos\phi_0}{2\pi Cs}\int_0^{2\pi} p[\theta(\psi)][\cos\phi_0 - \sin\phi_0\,\cot\mu\,\cos\psi]d\psi - \omega_e\,\cot\mu\,\sin\lambda$$

When $p(\theta)$ is a constant p_0, these equations simplify to

$$d\mu/dt = \omega_e\,\cos\lambda$$

$$\left\langle\frac{d\lambda}{dt}\right\rangle = -\frac{\delta}{2\pi H_0}\int_0^{2\pi}\left(\frac{S}{c}\right)A\,p_0[\cos\phi_0 - \sin\phi_0\,\cot\mu\,\cos\psi]d\psi - \omega\,\cot\mu\,\sin\lambda$$

or, averaging over ψ,

$$\left\langle\frac{d\lambda}{dt}\right\rangle = -\delta\left(\frac{S}{c}\right)\frac{A_s p_0}{H_0}\cos\phi_0 - \omega_e\,\cot\mu\,\sin\lambda$$

Since $H_0\cos\phi_0 = Cs$, this may be written as

$$\left\langle\frac{d\lambda}{dt}\right\rangle = -\delta\left(\frac{S}{c}\right)\frac{A_s p_0}{Cs}\cos^2\phi_0 - \omega_e\,\cot\mu\,\sin\lambda \quad (35)$$

The first term in Eq (35) is a secular term, whereas the second term (due to the rotation of $\mathbf{e}$) is periodic in μ and λ

§ The effect of direct polar radiation on a satellite in the shape of a rectangular prism has been studied by Ives[16]

The force due to direct solar radiation on a right circular cylinder of radius R and height h with surface reflectivity ρ for both its sides and endpieces is

$$\mathbf{F}_1 = -\delta(S/c)\{2Rh \sin\theta[(1 + \tfrac{1}{3}\rho)\mathbf{e} - \tfrac{4}{3}\rho \cos\theta\mathbf{k}] + \pi R^2 \cos\theta[H(\theta) - 2H(\theta - \pi/2)][(1 - \rho)\mathbf{e} + 2\rho \cos\theta\mathbf{k}]\}$$

Thus

$$F_1'(\theta) = (S/c)\{2Rh \sin\theta(1 + \tfrac{1}{3}\rho) + \pi R^2 \cos\theta(1 - \rho)[H(\theta) - 2H(\theta - \pi/2)]\} \quad (36)$$

in which $H(\theta)$ is the Heaviside unit step function Assuming that p is a constant p_0, one obtains

$$\mathbf{M}_1(\theta) = -\delta(S/c)\{2Rh \sin\theta(1 + \tfrac{1}{3}\rho) + \pi R^2 \cos\theta(1 - \rho)[H(\theta) - 2H(\theta - \pi/2)]\}p_0\mathbf{k} \times \mathbf{e}$$

and

$$V(\theta) = \delta\left(\frac{S}{c}\right)p_0\left\{Rh\left(1 + \frac{1}{3}\rho\right)\left[\theta - \frac{1}{2}\sin 2\theta\right] + \pi R^2(1 - \rho)\left[\frac{\sin^2\theta}{2}H(\theta) - 2\left(\frac{\sin^2\theta}{2} - \frac{1}{2}\right)H\left(\theta - \frac{\pi}{2}\right)\right]\right\}$$

and Eq (18) is still satisfied Therefore,

$$d\mu/dt = \omega \cos\lambda$$

$$\left\langle\frac{d\lambda}{dt}\right\rangle = -\frac{\delta p_0}{2\pi H_0}\int_0^{2\pi}F_1'[\theta(\psi)][\cos\phi_0 - \sin\phi_0 \cot\mu \cos\psi]d\psi - \omega \cot\mu \sin\lambda$$

where $F_1'(\theta)$ is given by Eq (36) From Eq (33),

$$\cos\theta = \mathbf{k} \quad \mathbf{e} = \cos\phi \cos\mu + \sin\phi \sin\mu \cos\psi$$

and

$$\sin\theta = (a \cos^2\psi + b \cos\psi + c)^{1/2}$$

where

$$a = -\sin^2\phi \sin^2\mu$$

$$b = -\tfrac{1}{2}\sin 2\phi \sin 2\mu$$

$$d = 1 - \cos^2\phi \cos^2\mu$$

The average precession rate is

$$\left\langle\frac{d\lambda}{dt}\right\rangle = -\delta\left(\frac{S}{c}\right)\frac{p_0}{2\pi H_0}\left[2Rh\left(1 + \frac{1}{3}\rho\right)I_1 + \pi R^2(1 - \rho)I_2\right] - \omega \cot\mu \sin\lambda$$

where

$$I_1 = \int_0^{2\pi}(a \cos^2\psi + b \cos\psi + d)^{1/2}[\cos\phi_0 - \sin\phi_0 \cot\mu \cos\psi]d\psi$$

$$I_2 = \int_0^{2\pi}2\pi\left\{H\left[\theta(\psi)\right] - 2H\left[\theta(\psi) - \frac{\pi}{2}\right]\right\} \times [\cos\phi_0 \cos\mu + \sin\phi_0 \sin\mu \cos\psi][\cos\phi_0 - \sin\phi_0 \cot\mu \cos\psi]d\psi$$

Reflected Solar and Direct Earth Radiation

From Eqs (8–10),

$$\mathbf{F}_2 = -\delta\int_{A_2^*}\int_{\bar{A}}\left(\frac{\bar{\rho}S}{c\pi L^2}\right)\cos\eta_1 \cos\xi_1[\cos^2\xi_2(1 + \rho)\mathbf{n} + \cos\xi_2(1 - \rho)\mathbf{n} \times (\mathbf{l} \times \mathbf{n})]d\bar{A}\; dA^*$$

$$\mathbf{F}_3 = -\int_{A_3^*}\int_A\left(\frac{\epsilon\sigma T^4}{c\pi L^2}\right)\cos\xi_1[\cos^2\xi_2(1 + \rho')\mathbf{n} + \cos\xi_2(1 - \rho')\mathbf{n} \times (\mathbf{l} \times \mathbf{n})]dA\; dA$$

where $\bar{A}_e$ is the surface area of the earth-atmosphere model sphere seen by dA and not in the earth's shadow, A_2^* is the total surface area of the satellite seen by that part of the model sphere not in the shadow, A is the surface area of the earth seen by dA, and A_3^* is the total surface area of the satellite seen by the earth

It has already been noted that these surface integrations are very complicated for an arbitrary position of an arbitrarily shaped satellite (even with the assumed geometric symmetry) The analytical description of $\mathbf{F}_2$ is especially complicated by the earth's shadow, for, even if the satellite does not pass through the shadow, the reflected solar radiation that the satellite receives would be influenced by it In order to make some statements about the effects of $\mathbf{F}_2$ and $\mathbf{F}_3$ upon the satellite's attitude, the following form will be assumed for both forces:

$$\mathbf{F}_{2\,3} = F_{2\,3}'(\theta')\mathbf{e} + F_{2\,3}''(\theta')\mathbf{k}$$

where the shadow factor δ in $\mathbf{F}_2$ has been dropped for the time being, and θ' is the angle between $\mathbf{e}$ and $\mathbf{k}$ Because of the assumed satellite symmetry, this form will be correct for $\mathbf{F}_3$ when the satellite's orbit about the earth is circular (otherwise it would also depend on r) It is correct for $\mathbf{F}_2$ only when the satellite is on the earth-sun line, but it will serve as an approximation for other positions of the satellite In general, it would also depend on r, θ, and the angle between $\mathbf{e}$ and $\mathbf{e}$

The moment about the satellite's center of mass will then be

$$\mathbf{M}_{2\,3} = p_{2\,3}(\theta')F_{2\,3}'(\theta')\mathbf{k} \times \mathbf{e} \quad (37)$$

where the separation distance p is also assumed to depend only on θ' The subscripts will now be dropped on $\mathbf{F}$, $\mathbf{M}$, etc , where it is to be understood that the results apply to reflected solar as well as to direct earth radiation The moment is assumed to be derivable from a potential so that $M(\theta') = -\partial V'/\partial\theta'$ If the analysis is restricted to satellite orbits that are circular, there are two integrals of the motion:

$$\mathbf{H} \quad \mathbf{k} = h_1' \text{ or } \omega = s(\text{const}) \quad (38)$$

and

$$\tfrac{1}{2}\mathbf{H} \quad \boldsymbol{\omega} - \mathbf{H} \quad \boldsymbol{\omega} + V'(\theta') = h_2' \quad (39)$$

These integrals are exactly the same as Eqs (17) and (19) with $\boldsymbol{\omega}_e$ (angular velocity of the earth about the sun) replaced by $\boldsymbol{\omega}_0$ (angular velocity of the satelite about the earth, constant for a circular orbit) Equation (39) leads to the following relation for H:

$$H = H_0\left\{1 + \frac{A\omega_0}{H_0}(\cos\nu' - \cos\nu_0') - \frac{A}{H_0^2}(V' - V_0') + O\left[\frac{A^2\omega_0}{H_0^2}\right] + O\left[\frac{A^2}{H_0^4}(V' - V_0')^2\right]\right\}$$

where H_0, V_0', and ν_0' are the initial or unperturbed values of H, V', and of ν', the angle between $\mathbf{H}$ and $\boldsymbol{\omega}_0$ Similarly to Eq (22), the case of interest is

$$(A\omega_0/H_0)(\cos\nu' - \cos\nu_0') - (A/H_0^2)(V' - V_0') \ll 1 \quad (40)$$

so that H may be replaced by its unperturbed value H_0 Note that if Eq (40) is satisfied Eq (22) is also satisfied because $\omega \ll \omega_0$ From Eq (38) $\mathbf{H} \quad \mathbf{k} = H \cos\phi = h_1'$, and so, if $H = H_0$, then $\phi = \phi_0$ The precession rate of $\mathbf{k}$ about $\mathbf{H}$ (see Fig 5) is $\dot{\psi}' = H/A$; thus, for $H = H_0$, $\dot{\psi}' = H_0/A$, which is its unperturbed precession rate There are no secular effects in the motion of the axis of symmetry about the angular momentum vector From Fig 5, $\mathbf{H} = H_0\mathbf{h}'$, where

$$\mathbf{h}' = \cos\mu'\mathbf{I} + \sin\mu'(\sin\lambda'\mathbf{J} + \cos\lambda'\mathbf{K})$$

The equation of motion about the satellite's center of mass is

$$-\sin\mu' \frac{d\mu'}{dt}\mathbf{I} + \cos\mu'(\sin'\lambda\mathbf{J} + \cos\lambda'\mathbf{K})\frac{d\mu'}{dt} +$$
$$\sin\mu'(\cos\lambda'\mathbf{J} - \sin\lambda'\mathbf{K})\frac{d\lambda'}{dt} + \cos\mu'(\boldsymbol{\omega}_p \times \mathbf{I}) +$$
$$\sin\mu'[\sin\lambda'(\boldsymbol{\omega}_p \times \mathbf{J}) + \cos\lambda'(\boldsymbol{\omega}_p \times \mathbf{K})] = \frac{\mathbf{M}(\theta')}{H_0} \quad (41)$$

when written in terms of the perigee coordinate system (XYZ) This coordinate system, which has an angular velocity $\boldsymbol{\omega}_p$ relative to inertial space, caused by changes in the orbital elements as a result of orbit perturbations, is defined as follows The perigee reference (XYZ) is a non-inertial coordinate system with origin at the satellite's center of mass; the ZX plane is the orbit plane, with the Z axis parallel to the position vector at perigee and the X axis parallel to the tangent to the orbit at perigee The associated unit vectors are **I, J, K** Accordingly,

$$\boldsymbol{\omega}_p = \omega_{p1}\mathbf{I} + \omega_{p2}\mathbf{J} + \omega_{p3}\mathbf{K}$$
$$\omega_{p1} = \sin\imath\cos\omega(d\Omega/dt) - \sin\omega(d\imath/dt)$$
$$\omega_{p2} = \cos i(d\Omega/dt) + (d\omega/dt)$$
$$\omega_{p3} = \sin i\sin\omega(d\Omega/dt) + \cos\omega(di/dt)$$

in which i, ω, Ω are the usual orbital elements Equation (41) may be written, using Eq (37), as

$$\frac{d\mu'}{dt} = -\frac{F'(\theta')p(\theta')}{H_0\sin\mu'}\cos f\mathbf{k}\ \ \mathbf{J} - \omega_{p3}\sin\lambda' + \omega_{p2}\cos\lambda' \quad (42)$$

$$\frac{d\lambda'}{dt} = \frac{F'(\theta')p(\theta')}{H_0\sin\mu'}[\cos\lambda'(\sin f\mathbf{K} - \cos f\,\mathbf{I}) +$$
$$\sin\lambda'\sin f\,\mathbf{J}]\ \ \mathbf{k} + \omega_{p1} - \cot\mu'[\omega_{p3}\cos\lambda' + \omega_{p2}\sin\lambda'] \quad (43)$$

where f is the true anomaly, and, from Fig 5,

$$\mathbf{k} = (\cos\phi\cos\mu' + \sin\phi\sin\mu'\cos\psi')\mathbf{I} +$$
$$(\cos\phi\sin\mu'\sin\lambda' + \sin\phi\cos\lambda'\sin\psi' -$$
$$\sin\phi\cos\mu'\sin\lambda'\cos\psi')\mathbf{J} + (\cos\phi\sin\mu'\cos\lambda' -$$
$$\sin\phi\sin\lambda'\sin\psi' - \sin\phi\cos\mu'\cos\lambda'\cos\psi')\mathbf{K}$$

In order to determine the secular effects in Eqs (42) and (43), it is necessary to replace the instantaneous moments by the average moments This is achieved by averaging the perturbing moments over a complete rotation of **k** about **H** (ψ' ranging from 0 to 2π) and over a complete revolution of the satellite about the earth (f ranging from 0 to 2π) The values of $d\omega/dt$, di/dt, etc , to be substituted in Eqs (42) and (43) are the average values and thus will not be averaged here The average rates of change of μ' and λ' are

$$\left\langle\frac{d\mu'}{dt}\right\rangle = -\frac{1}{4\pi^2}\int_0^{2\pi}\int_f \frac{F'[\theta'(\psi')]p[\theta'(\psi')]}{H_0\sin\mu'}\cos f(\mathbf{k}\ \ \mathbf{J}) \times$$
$$df d\psi' - \omega_{p3}\sin\lambda' + \omega_{p2}\cos\lambda' \quad (44)$$

$$\left\langle\frac{d\lambda'}{dt}\right\rangle = \frac{1}{4\pi^2}\int_0^{2\pi}\int_f \frac{F'[\theta'(\psi')]p[\theta'(\psi')]}{H_0\sin\mu'}[\cos\lambda'(\sin f\mathbf{K} -$$
$$\cos f\,\mathbf{I}) + \sin\lambda'\sin f\,\mathbf{J}]\ \ \mathbf{k}\,df d\psi' +$$
$$\omega_{p1} - \cot\mu'[\omega_{p3}\cos\lambda' + \omega_{p2}\sin\lambda'] \quad (45)$$

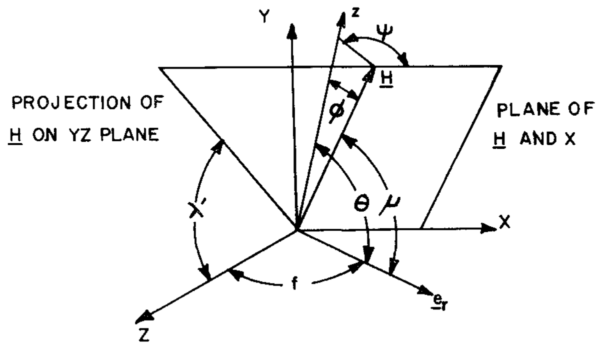

Fig 5 Orientation of angular momentum vecto1 and axis of symmetry of satellite relative to the perigee coordinate system (*X*, *Y*, *Z*)

At this point the effects of each force, $\mathbf{F}_2$ and $\mathbf{F}_3$, can be analyzed separately In the case of direct earth radiation, the average on f is from 0 to 2π, because the earth's shadow has no effect From the preceding equations, one finds

$$\langle d\mu'/dt\rangle = -\omega_{p3}\sin\lambda' + \omega_{p2}\cos\lambda' \quad (46)$$
$$\langle d\lambda'/dt\rangle = \omega_{p1} - \cot\mu'[\omega_{p3}\cos\lambda' + \omega_{p2}\sin\mu'] \quad (47)$$

because the effects of the perturbing moment $\mathbf{M}_3$ are completely periodic in the true anomaly The satellite's motion for direct earth radiation under the stated assumptions can be described, therefore, as an unperturbed motion of the axis of symmetry about the angular momentum vector, which itself remains practically constant in magnitude but changes its orientation relative to (XYZ) as a result of orbit perturbations only

In the case of reflected solar radiation, there are two possibilities: 1) satellite never enters the earth's shadow, and 2) satellite enters the earth's shadow at $f = f_1$ and leaves the shadow at $f = f_2$ In the first possibility, the effect of the perturbing moment $\mathbf{M}_2$ is completely periodic in f and, averaging over f from 0 to 2π, reproduces the same result as Eqs (46) and (47) In the second possibility, when the satellite enters the shadow, $\mathbf{F}_2 = 0$, so that the average over f is not zero even though the effect is still periodic in f From Eqs (44) and (45), one obtains

$$\left\langle\frac{d\mu'}{dt}\right\rangle = -\frac{1}{4\pi^2}\frac{\sin f_1 - \sin f_2}{H_0\sin\mu'}\int_0^{2\pi}F_3'[\theta'(\psi')]p[\theta'(\psi')] \times$$
$$(\mathbf{k}\ \ \mathbf{J})d\psi' - \omega_{p3}\sin\lambda' + \omega_{p2}\cos\lambda' \quad (48)$$

$$\left\langle\frac{d\lambda'}{dt}\right\rangle = -\frac{1}{4\pi^2 H_0\sin\mu'}\int_0^{2\pi}F_3'[\theta'(\psi')]p[\theta'(\psi')] \times$$
$$\{\cos\lambda'[(\cos f_1 - \cos f_2)\mathbf{K} + (\sin f_1 - \sin f_2)\mathbf{I}] +$$
$$\sin\lambda'(\cos f_1 - \cos f_2)\mathbf{J}\}$$
$$\mathbf{k}d\psi' + \omega_{p1} - \cot\mu'[\omega_{p3}\cos\lambda' + \omega_{p2}\sin\lambda'] \quad (49)$$

where μ' is considered constant during the integration These integrals cannot be worked out completely until particular forms for $F_3'(\theta')$ and $p(\theta')$ are specified The satellite's motion due to reflected solar radiation is, therefore, an unperturbed motion of the axis of symmetry about the angular momentum vector, which remains approximately constant in magnitude and whose orientation relative to (XYZ) is given by Eqs (46) and (47) if the satellite does not pass through the earth's shadow, or by Eqs (48) and (49) for satellite eclipses

The angular velocity of (XYZ) relative to inertial space, $\boldsymbol{\omega}_p$, depends on orbit perturbations, and for any particular satellite orbit the important perturbations will have to be worked out by some appropriate technique in order to determine $d\Omega/dt$, $d\omega/dt$, and $d\imath/dt$

It is well known, for instance, that the oblateness of the earth produces, among other effects, a secular increase in both Ω and ω (see Kozai[15] as an example):

$$d\Omega/dt = -(2\pi\epsilon\bar{R}\,^2/p^2P)\cos i$$

$$d\omega/dt = (\pi\epsilon\bar{R}\,^2/p^2P)(5\cos \quad - 1)$$

where p is the focal parameter of the orbit, $\bar{R}_e$ the equatorial radius of the earth, P the orbital period, i the inclination of the orbit to the equatorial plane, and ϵ a dimensionless parameter that characterizes the oblateness of the earth (ϵ = 1 6331 $\times 10^{-3}$)

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Scattering and Absorption of Plane Waves by Cylindrically Symmetrical Underdense Zones

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Scattering and absorption of plane electromagnetic waves by cylindrically symmetrical, but axially and radially inhomogeneous, nearly transparent zones are computed in the Born approximation Scattered fields and the cross sections are presented in analytical form in terms of the elementary angles and an integral over the scattering zone The results are intended for application to meteorite and missile trail scattering; factors for the scattered amplitude are presented analytically and graphically for several electron-concentration distributions The role of turbulence is discussed, and the locally homogeneous but anisotropic turbulence case is solved for several arbitrary correlation functions It is shown that under rather loose conditions the turbulent contribution to the scattering cross section can exceed that due to the mean distribution, and that even weak anisotropy can introduce significant aspect angle sensitivity into the cross-sectional values

Introduction

THE scattering of electromagnetic waves by inhomogeneous regions has received considerable attention in recent years, with particular emphasis on cylindrical[1–4] and spherical[5–7] zones Motivation for such work stems from the investigation of meteorite trails by radar,[8] the use of scatter from ionized zones as an aid to radio communication, and the scattering of waves by the trails of hypersonic vehicles [9]

The analysis of such scattering processes and explanation of observed data are by no means closed subjects [10–12] There is some evidence that magnetic anisotropy even may be important in the scattering from meteor trails at lower frequencies[13] (a complication which readily can be included in the method of analysis here) [14] A general procedure for handling inhomogeneous scattering problems is clearly needed, and this extension of previous work[4] constitutes another step in the exploitation of the Born approximation

By reformulating the problem of the cylindrical scatterer to include axial gradients in "dielectric constant," the prob

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